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Multi-dimensional non-LTE Transfer of Polarized Radiation
in Magnetic Fluxtubes

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Summary

Calculations of multi-dimensional transfer of the Stokes vector in magnetic fluxtubes are presented. The Zeeman splitting caused by the magnetic field modifies the multi-dimensional effects in different ways. The line opacities are changed, which affects the number of photons from the hot fluxtube walls that can reach the fluxtube axis. Similarly the probabilities of photon escape in various directions and at various frequencies are changed. These effects are considered in detail for all the Stokes parameters.

It is shown how the multi-dimensional effects influence the relation between the apparent field strength observed with a solar magnetograph and the true field strength. The line-ratio method, used to derive true field strengths from simultaneous magnetograph recordings in two spectral lines of different Landé factor is tested. It is found that this method is insensitive to all the various model parameters, like collisional excitation cross section, fluxtube radius, or Wilson depression, provided that the line pair is properly chosen, using equally strong lines from the same multiplet, e.g. Fe I $\lambda\lambda 525.0$ and 524.7 nm. Observations in single lines with Babcock-type magnetographs are not suitable for deriving field strengths, since the results are very model dependent.

Key words: multi-dimensional - non-LTE - polarization - magnetic field - sun

1. Introduction

In an earlier paper (Stenholm and Stenflo, 1977) we explored the effects of multi-dimensional radiative transfer in magnetic fluxtubes for spectral lines with zero Landé factor. It was shown how line weakenings develop due to channelling of the radiation from the hot walls of the fluxtubes when the field is confined by gas pressure. In the past, when multi-dimensional effects have been ignored, such line weakenings have been interpreted in terms of temperature enhancement in the magnetic-field regions (facular models).

In the present paper we extend these calculations to treat the full polarization for lines with an arbitrary Landé factor. The multi-dimensional effects on the various Stokes parameters and their contributions to the line source function are considered in detail.

Such calculations can be used to study how the various non-LTE parameters (e.g. collisional excitation cross section, oscillator strength, geometry of fluxtube) influence the relation between the apparent field strength observed with a solar magnetograph and the true field strength. In particular the line-ratio method developed by Stenflo (1973, 1975, 1976a) to derive true field strengths from simultaneous magnetograph observations in two spectral lines of different Landé factor can be tested. The calculations show what the errors in the field strength will be if the line-ratio observations are interpreted in terms of an LTE analysis.

2. Calculations

2.1 Non-LTE Theory Used

To carry out our calculations, we have developed a non-LTE formalism, which is basically a reformulation of the theory of Domke and Staude (1973). The principal assumptions are: 1. Complete redistributions among the Zeeman sublevels, with no coherence effects. 2. Complete redistribution in frequency.

The start form of the transfer equation is

$$\frac{d\tilde{I}_v}{ds} = -\kappa_c (\tilde{I}_v - B_v \underline{1}) - \kappa_L \tilde{I}_v + \underline{j}_{ic} + \underline{j}_{th}, \quad (1)$$

where $\tilde{I}_v$ is the Stokes vector

$$\tilde{I}_v = \begin{pmatrix} I_v \\ Q_v \\ U_v \\ V_v \end{pmatrix}, \quad (2)$$

and $\underline{1}$ is a unit vector (first position unity, remaining position zero). The continuum source function is thus assumed to be given by the Planck function B_v and is unpolarized. κ_c is the continuum opacity, κ_L the line opacity matrix.

$$\kappa_L = \kappa_0 \underline{K}, \quad (3)$$

where

$$\kappa_0 = (n_i B_{ij} - n_j B_{ji}) \frac{h\nu}{4\pi} \quad (4)$$

and

$$\tilde{K} = \begin{pmatrix} K_I & K_Q & K_U & K_V \\ K_Q & K_I & 0 & 0 \\ K_U & 0 & K_I & 0 \\ K_V & 0 & 0 & K_I \end{pmatrix}. \quad (5)$$

Defining

$$\varphi_B = \frac{1}{2} [\varphi_0 - \frac{1}{2} (\varphi_+ + \varphi_-)] (1 - \mu^2), \quad (6)$$

we get

$$K_I = \varphi_B + \frac{1}{2} (\varphi_+ + \varphi_-),$$

$$K_Q = \varphi_B \cos 2\chi,$$

$$K_U = \varphi_B \sin 2\chi, \quad (7)$$

and

$$K_V = \frac{1}{2} (\varphi_- - \varphi_+) \mu.$$

μ is the cosine of the angle between the magnetic field and the line of sight, and χ is the azimuth angle of the magnetic-field vector in the counter-clockwise direction relative to the positive Q direction. φ is the redistribution function, normalized to unit area, assumed to be gaussian. Index + denotes that the profile is

shifted towards smaller frequencies by the amount

$$\Delta\nu_B = 1.401 \times 10^{10} g B, \quad (8)$$

where B is the magnetic field strength in Tesla (T), and g is the Landé factor. Similarly, index $-$ denotes shift towards larger frequencies by the amount (8), while index 0 denotes an unshifted profile.

Scattering in the $i - j$ transition (radiative absorption from level i to level j with subsequent spontaneous emission from level j to level i) is taken care of by the incoherent scattering term j_{ic} . All other emission is assumed to occur in LTE and is described by the j_{th} term (th for 'thermal'). This formalism is analogous to that used in the theory of resonance-line scattering by Stenflo (1976b).

$$j_{th} = j_{th} \frac{K}{1}, \quad (9)$$

where

$$j_{th} = (1 - \alpha) n_j A_{ji} \frac{h\nu}{4\pi}, \quad (10)$$

and

$$\alpha = \frac{n_i B_{ij} \int \varphi_\nu J_\nu d\nu}{\sum_k n_k R_{kj}} \quad (11)$$

represents the fraction of spontaneous emissions from level j to level i which have been immediately preceded by radiative excitation from i to j (cf. Stenflo, 1976b). J_ν is the mean intensity, and R_{kj} is the total transition rate from level k to level j (sum of radiative and collisional transitions). n_j , A_{ji} , etc., have their usual meaning.

The incoherent scattering term can be written

$$\tilde{j}_{ic} = \sigma \int_0^\infty \int \tilde{F} \tilde{I}_\nu d\nu \frac{d\omega}{4\pi}, \quad (12)$$

where $\tilde{F}$ describes the redistribution in frequency, angle, and polarization. The total scattering cross-section is

$$\sigma = n_i B_{ij} \frac{h\nu}{4\pi} \frac{n_j A_{ji}}{\sum_k n_j R_{jk}} \quad (13)$$

(cf. Stenflo, 1976b).

To obtain a simple expression for the redistribution matrix $\tilde{F}$ we assume complete redistribution of the electrons between the Zeeman sublevels. This should be a very good approximation throughout the photosphere, where the collisional coupling between closely spaced levels like the Zeeman sublevels must be very strong. If we further assume complete redistribution in frequency, we obtain

$$\tilde{F} = (\underline{K} \quad \underline{1}) \begin{pmatrix} \underline{1}^T \\ \underline{K}' \end{pmatrix}. \quad (14)$$

$\underline{1}^T$ is the transpose of $\underline{1}$. $\underline{K}$ can be regarded as the sum of the various $\underline{K}_m$, where index m refers to Zeeman sublevel m . (14) means that absorption to a given Zeeman sublevel m can be followed by reemission from any other sublevel without reference to level m .

Using (14), (12), and (2), we obtain

$$\underline{j}_{ic} = \sigma e \underline{K} \underline{1}, \quad (15)$$

where

$$e = \oint_0^\infty (\underline{K}_I \underline{I}_v + \underline{K}_Q \underline{Q}_v + \underline{K}_U \underline{U}_v + \underline{K}_V \underline{V}_v) dv \frac{d\omega}{4\pi}. \quad (16)$$

With the standard definitions

$$S_{ji} = n_j A_{ji} / (n_i B_{ij} - n_j B_{ji}) \quad (17)$$

(usual line source function in the absence of polarization),

and

$$r = \kappa_c / \kappa_0 \quad (18)$$

(ratio between continuum and line opacities), we can rewrite (1)

to obtain the transfer equation

$$\frac{d\underline{I}_v}{d\tau_v} = \frac{\underline{K} + r \underline{E}}{\underline{K}_I + r} \underline{I}_v - \frac{\underline{K} \underline{1}}{\underline{K}_I + r} S_L - \frac{r \underline{1}}{\underline{K}_I + r} B_v, \quad (19)$$

where we have introduced an optical depth scale defined by

$$d\tau_v = -\kappa_0 (K_I + r) ds \quad (20)$$

and a line source function S_L given by

$$S_L = S_{ji} \left(\frac{\alpha e}{\int \varphi_v J_v dv} + 1 - \alpha \right). \quad (21)$$

$\underline{E}$ in (19) is a 4×4 unity matrix, with first position (E_{11}) unity, the remaining positions zero. α and e in (21) are given by (11) and (16).

For a two-level atom we have

$$S_{ji} = (\int \varphi_v J_v dv + \epsilon B_v) / (1 + \epsilon), \quad (22)$$

$$\epsilon = \frac{C_{ji}}{A_{ji}} (1 - e^{-h\nu/kT}), \quad (23)$$

and

$$\alpha = \int \varphi_v J_v dv / (\int \varphi_v J_v dv + \epsilon B_v). \quad (24)$$

Inserting these expressions in (21), we obtain the line source function.

$$S_L = (e + \epsilon B_v) / (1 + \epsilon). \quad (25)$$

2.2. Application of the Core-saturation Method

To solve the vector transfer equation in non-LTE for the multi-dimensional case we use the method of core saturation first proposed by Rybicki (1971) and later extended to the multi-

dimensional case by Stenholm (1977a). The method was used by Stenholm and Stenflo (1977) for multi-dimensional calculations of unpolarized radiation in fluxtube geometries.

The main feature of the core-saturation method is to precondition the transfer equation to allow it to be solved by simple methods, e.g. by lambda iteration. This leads to simple computer programs which are fast and highly adaptable. Accordingly it was fairly straightforward to adapt the computer code used for the calculations of Stenholm and Stenflo (1977) to the case of the full vector equation (19) in the presence of Zeeman-effect polarization.

The physical basis for the preconditioning is the following: When the opacity becomes very large, as in the core of a spectral line, the depth-gradient of I_v becomes very small, and (19) can be approximated by

$$(\underline{K} + r\underline{E}) \underline{I}_v = \underline{K} \underline{1} S_L + r \underline{1} B_v. \quad (25)$$

Accordingly we can divide the scattering term in S_L in one passive saturated core term and one unsaturated wing term, which includes all interactions with distant points.

After some algebraic reductions (cf. Stenholm, 1977b) we obtain the line source function for a two-level atom in the more suitable form

$$S_L = \left[\int_w \oint (K_I I_v + K_Q Q + K_U U_v + K_V V_v) \frac{d\omega}{4\pi} dv + \epsilon B_v + \delta_c B_v \right] / (P_e + \delta_c + \epsilon). \quad (27)$$

w under the integral sign indicates that the integration is made over the wings only. The escape probability P_e is given by

$$P_e = \frac{1}{3} \int_w (\varphi_0 + \varphi_+ + \varphi_-) dv \quad (28)$$

and the contribution from the continuum by

$$\delta_c = \frac{1}{2} \int_c \int_{-1}^1 \xi d\mu dv, \quad (29)$$

where

$$\xi = r [K_I (K_I + r) - K_Q^2 - K_U^2 - K_V^2] / \quad (30)$$

$$[(K_I + r)^2 - K_Q^2 - K_U^2 - K_V^2]. \quad (31)$$

c under the integral sign indicates that the integration is made over the core only.

The next step is to determine the critical frequency $\nu_{\min}$ separating the core and the wings, for every grid point in the atmosphere. This is done by solving the equation

$$\tau_{\nu_{\min}} = \gamma \geq 1. \quad (32)$$

$\nu_{\min}$ is determined with respect to the direction, for which

the interaction between the given point in the atmosphere and the boundary is the largest. In the case of isotropic emission this means the direction of highest probability for photon escape, i.e. smallest absorption probability. In the presence of magnetic fields, the emission is however highly anisotropic. In this case, the direction of largest interaction with the boundary is determined by the maximum of the product of the emission probability E_v and the probability of escape at the boundary $\exp(-\tau_v)$. Both E_v and τ_v vary strongly with direction.

$$E_v = K_I + r, \quad (33)$$

with K_I given by (6) and (7), and r by (18) and (4). Note that due to the complicated shape of the profile K_I we can have more than one core and thus more than one critical frequency at some points in the atmosphere for some field strengths.

γ in (32) is a free parameter which is chosen to optimize accuracy and convergence in the numerical calculations.

2.3. Model atmosphere

To separate the multi-dimensional effects from the temperature effects in the fluxtubes, we will use the HSRA (Gingerich et al., 1971) both inside and outside the fluxtube. The only difference is that the zero point of the geometrical height scale is shifted downwards inside the fluxtube by 150 km. Thus LTE calculations in the absence of magnetic fields should give identical line profiles inside and outside the fluxtubes.

To confine a field of 0.2 T a Wilson depression of about 150 km is needed (Stenflo, 1977). Typical radii of fluxtubes in quiet regions should be 100 km (Stenflo, 1973, 1976a). These values of Wilson depression and radius were used as nominal values in the model of Stenholm and Stenflo (1977), and will be used in the present calculations as well. The Wilson depression is assumed to be independent of radial distance R from the cylinder axis out to 99 km, where we have a 1 km thick transition layer. This layer is optically thin in most of the line-forming region. The total line broadening is assumed to be height independent, with a constant broadening velocity of 2 km s^{-1} .

The field strength B inside the fluxtube has been varied to generate a number of model atmospheres differing only in B . For the line-formation problem, this is equivalent to saying that we keep the field strength fixed at e.g. 0.2 T but instead vary the Landé factor of the line studied. The aim of the paper is to study multi-dimensional effects in the presence of magnetic fields, not to construct improved fluxtube models.

The magnetic field is assumed to be height independent, vertical, and constant out to $R = 100 \text{ km}$, zero beyond. Assuming a Landé factor of 3.0, the field strengths used in the calculations are 0, 0.025, 0.05, 0.1, 0.15, and 0.2 T.

2.4. Model Atom

We have used a two-level model atom. It may be regarded as a neutral iron atom with all levels in LTE except the levels

of one resonance transition, corresponding to the line considered. The ionization equilibrium is assumed to follow that of iron. Our resonance line has a wavelength of 500 nm, an oscillator strength $f = 2.0 \times 10^{-5}$, a Landé factor $g = 3$, and a nominal collisional excitation cross section in units of the Bohr cross section $Q = 0.1$. These values of f , g , and Q are valid for the commonly used magnetograph line Fe I $\lambda 525.0$ nm (Lites, 1972). The background continuum is assumed to be given by the Planck function. To simulate the behaviour of the 525.0 nm line, the element abundance has been adjusted so that the calculated line depth at line center outside the fluxtubes agrees with the observed line depth for the 525.0 nm line. To study how the results depend on Q and f , we have also made calculations for $Q = 0.01$ and $Q = 0.3$ with f nominal, as well as for $f = 2.0 \times 10^{-4}$ with Q nominal.

More details about the model atom used are given in Stenholm and Stenflo (1977). In that paper we stated that the oscillator strengths used were 2.5×10^{-5} and 2.5×10^{-4} , respectively. This is a mistake, however, since we actually used 2.0×10^{-5} and 2.0×10^{-4} , precisely as in the present paper.

Results for other values of the Landé factor can be obtained directly from our calculations with different values of the field strength with the nominal Landé factor of three, since the results do not depend separately on field strength or Landé factor but on the product, the Zeeman splitting. Thus our results are valid for a line with Landé factor g' if the field strengths are scaled from B to B' , where

$$B' = 3 B / g'. \quad (34)$$

2.5. Quadrature and Convergence

The accuracies of different quadratures have been determined by calculating the line profiles in the vertical direction. Accordingly we have found that a choice of 32 - 38 height points, 6 radial points (the last so far out that it corresponds to the plane-parallel case), 2 azimuth angles, and 3 altitude angles is adequate and gives an error that rarely exceeds 2 percent in the absolute values of I_v and V_v . For absolute values of V_v which are less than 2 % of I_v , the error in V_v tends to grow slowly to values above the mentioned relative error of 2 %. Note that this choice of angle quadrature means that $\tilde{I}_v$ is calculated in $2 \times 2 \times 3 = 12$ directions at every point of the atmosphere (cf. Stenholm, 1977a).

The values of γ used with the above quadrature is 4. This ensures fast convergence of the solution, with typically only 4 iterations.

The computer code with explanations is given in Stenholm (1977b).

3. Results

3.1. Line Profiles

The line profiles calculated in the vertical direction for the four lines considered are given in Figures 1 - 4. When there are significant variations with radial distance R inside the fluxtube, profiles at $R = 99$ km are shown in addition to the profiles at

$R = 0$ and $R = \infty$. For comparison, the LTE profiles inside and outside the fluxtube are also given. Only the results for four values of the field strength, 0.025, 0.05, 0.1, and 0.2 T are illustrated.

We immediately recognize the line weakening in the non-LTE profiles inside the fluxtube, which was extensively discussed by Stenholm and Stenflo (1977) for lines with zero Landé factor. This line weakening is due to the influx into the more transparent fluxtube of photons from the surrounding hot walls, which are present due to the Wilson depression. This causes the line source function to rise above its LTE value inside the cylinder. With an LTE interpretation, such line weakenings correspond to a fictitious temperature enhancement approximately equal to that derived in recent LTE facular models (cf. Stenholm and Stenflo, 1977). We also see in Figures 1 - 4 that the line weakening depends strongly on Q and f , because the collisional thermalization parameter ϵ in (25) and (23), which determines the amount of collisionally created photons relative to scattered photons, is proportional to Q/f .

The V profile is found to exhibit the same general pattern as the I profile, that is a weakening relative to LTE values. In most cases, the relative amount of weakening is equal in I and V . The weakening in V can be explained by a decrease of the depth gradient of the line source function, or, equivalently when B is small, by a decrease of the line-profile gradient dI_v / dv , which is proportional to V for small B .

For the cores of scattering-dominated lines, e.g. when $Q = 0.01$ and particularly for the stronger line with $f = 2.0 \times 10^{-1}$,

the general trend of a decreased absolute value of V with increasing line weakening tends to be reversed when $B \lesssim 0.1$ T. This has to do with the influence of line weakening on the shape of the core profiles. The gradient dI_v/dv in the core is larger when the line weakening is greater. The line weakening tends to reduce the saturation in the (trough-shaped) line profiles in the core. The cause of the reduced line-core saturation with accompanying increase in V can be traced back to the particular behaviour of the depth-variation of the line source function S_L around a line optical depth of unity, where much of the radiative coupling between the interior and exterior of the fluxtube occurs. This coupling increases with decreasing ϵ .

Comparing the non-LTE profiles for $R = 0$ and $R = 99$ km we note that the radial variation of I inside the cylinder increases with increasing amount of scattering. The explanation is as follows: When scattering increases, we get a more and more non-local situation with an increasing portion of the source function being caused by photons from the hot walls. Since the cylinder is not entirely optically thin in the radial direction, some photons will never reach the cylinder axis. More scattering will thus lead to an increased radial dependence of the source function. For a cylinder of 100 km radius the horizontal optical distance from axis to cylinder wall $\tau_R \approx 1$ at the height where $\tau_z \approx 1$ (cf. Stenholm and Stenflo, 1977).

3.2. Field Determinations Using Single Lines

For small values of the field strength, V_v is proportional to the intensity gradient $\partial I_v / \partial v$ and the longitudinal field strength B . Calibration of solar magnetographs essentially means a determination of $\partial I_v / \partial v$ so that the observed values of V_v can be converted to field strengths B_{obs} by simple multiplication by a constant calibration factor. As the field regions cannot be spatially resolved, $\partial I_v / \partial v$ is measured for the average line profile, which is representative of the non-magnetic regions far from the fluxtube. B_{obs} will therefore differ from B due to line weakenings in the fluxtubes and because the assumption of weak fields is incorrect.

Using our model, we can calculate the relation between B_{obs} and B (true longitudinal field strength) and study how it depends on Q , f , and position in the line profile. The gradient $\partial I_v / \partial v$ is calculated for the plane-parallel case outside the cylinder with $B = 0$. B_{obs} is then obtained from V_v by

$$B_{\text{obs}} = V_v / (1.401 \times 10^{10} \text{ g } \partial I_v / \partial v) \quad (35)$$

(cf. Stenflo, 1973). As the apparent field is derived from the average of V_v over the fluxtubes, it is actually

$$\bar{B}_{\text{obs}} = \frac{1}{\pi R_0^2} \int_0^{R_0} B_{\text{obs}} 2\pi R \, dR \quad (36)$$

that has to be compared with B . R_0 is the cylinder radius = 100 km.

In Figures 5 - 6 we have given $\bar{B}_{\text{obs}}/B$ as a function of B for $\Delta\nu = 0.25, 1.0$, and $2.0 \Delta\nu_D$. The ratio $\bar{B}_{\text{obs}}/B$ is generally constant out to $B \approx 0.025$ T. The departure from unity of the zero point of the $\bar{B}_{\text{obs}}/B$ curve and the frequency dependence of this departure is caused by the difference in $\partial I_\nu/\partial\nu$ between the inside and the outside of the fluxtube. This difference increases with increasing amount of scattering.

Due to the Zeeman saturation, the correction factors needed to transform $\bar{B}_{\text{obs}}$ to B grow rapidly with B for $B > 0.025$ T. These correction factors also depend strongly on Q and f . Thus the correction factors for a given frequency are very model dependent, primarily due to the poor knowledge of the collisional excitation cross section Q for Fe lines. In principle one can make the interpretation less model dependent by observing at a number of frequencies throughout the line profile and derive a model that gives the same value of B for all frequencies. If the correct model atmosphere were known, one could in principle even determine Q from observations, but this would require a knowledge of B as well.

Since we cannot assume that both the fluxtube model and Q are known, we strongly recommend not to use single-line observations with Babcock-type magnetographs for measuring magnetic field strengths.

In this context we want to note that our model gives a simple explanation of the peculiar observation by Livingston (1976) that the apparent field strengths are larger near the core of the Fe I 523.3 nm line than out in the wings. With a conventional LTE interpretation this would appear to indicate that the field strength

increases with height, which would contradict any concept of field confinement. The 523.3 nm line has an effective Landé factor of 1.3 and is considerably broader than even the stronger line used in our calculations (due to our neglect of the damping width). The field strengths in our diagrams should thus be scaled to much higher values to apply to this line. Accordingly the observations in the 523.3 nm line are qualitatively described by the diagram to the right of Figure 6, if we use the left part of that diagram, where the curves become horizontal. This part of the diagram shows precisely the effect observed by Livingston, i.e. a decrease of $\bar{B}_{\text{obs}}$ by approximately a factor of 2 when going from the core to the wings of the line. This decrease has nothing to do with the height variation of the field but is a multi-dimensional effect due to channelling of the radiation from the hot walls of the fluxtube. The correction factor needed to transform $\bar{B}_{\text{obs}}$ to B is largest in the line core, but becomes quite small in the wings.

3.3. Field Determinations Using Line Ratios

The method of line ratios was developed by Stenflo (1973, 1975, 1976a) to circumvent the model dependence previously discussed. The main idea is to use simultaneous observations in lines from the same multiplet, which have line parameters that should ideally be identical except for the Landé factor. This would result in identical source functions except when the magnetic field is very strong and thus remove the effect of scattering and other model dependence.

Using (34) we can directly obtain $\bar{B}_{\text{obs}_2}$, the field observed with a line of Landé factor = 2, from our calculations of $\bar{B}_{\text{obs}_3}$ valid for a line with a Landé factor = 3. Figures 7 and 8 illustrate different cases for $\bar{B}_{\text{obs}_3}/\bar{B}_{\text{obs}_2}$ as a function of frequency for $B = 0, 0.05, 0.1, 0.15$, and 0.2 T.

Let us first consider the case of two lines with identical line parameters (except S_L), which is illustrated to the upper left of Figure 7 and to the left of Figure 8. It is clear that the line ratios are quite insensitive to the amount of scattering (ϵ). Although a small dependence on ϵ is present, it would not significantly affect field-strength determinations except when $|\Delta\nu| < 1.5 \Delta\nu_D$ for the stronger line (Figure 8). This ϵ -dependence is ~~caused by~~ ^{due to} optical-depth effects ^{causing} ~~on~~ the variation of S_L with B (see Section 3.4). These effects are more pronounced when a large contribution to S_L comes from non-local photons. Comparing Figures 7 and 8 we also note that the line ratios vary strongly with f when changing f with a factor of 10. Since the f values are often known within 15 % or better, this will cause no trouble.

Let us now compare with the case when the lines have different source functions S_L due to different values of Q (remaining parts of Figures 7 and 8). As expected, the discrepancies grow rapidly with increasing difference in Q . Because of this dependence, Q has to be known with higher and higher accuracy when Q decreases. In the case of the weaker line, however, we find that the curves agree much better for higher values of B .

Also for the stronger line the errors in B with an LTE analysis are reasonably small when the Q ratio is as large as 100, but we also note that the region around $1.5 \Delta v_D$ can only partly discriminate between fields in excess of 0.1 T. To avoid this saturation effect for strong lines, one should use positions further away from the line center.

The rise above unity near the line center has to do with the trough-shaped form of the I_ν profile.

Stenflo (1973) used the two lines at 525.0 and 524.7 nm with Landé factors of 3 and 2, respectively. Both belong to multiplet no. 1 of Fe I and have the same $g_i f_{ij}$ values. The opacity structure should therefore be the same in both lines. Q has been estimated to be about 0.1 for the 525.0 nm line (Lites, 1972). Although the Q values are poorly known, it is reasonable to assume that Q should be about the same for the 524.7 nm line. ϵ will however be different for the two lines, since the ratios of g_i between the 524.7 and 525.0 nm lines is 5, and thus $f_{525.0}/f_{524.7} \approx 5$. This gives $\epsilon_{524.7} \approx 5 \epsilon_{525.0}$. This difference in ϵ is however quite small as compared with many of the cases that we have discussed. (Let us recall that when Q in Figures 7 and 8 is changed, it is assumed that f is fixed. In this case the Q ratio is the same as the ϵ ratio.) The diagrams to the upper right and lower left of Figure 7 give a feeling for what the effects may be. Accordingly, if the non-LTE curves represent the true situation but the interpretation of the observations is made according to LTE, the field strength

will be somewhat overestimated. Using the region around $1.5 \Delta v_D$, the field would be overestimated by $0.025 - 0.05$ T if the true field strength B is ≥ 0.1 T. There would appear to be no fields weaker than about 0.07 T. If however the line ratio is determined not only at one frequency but for various positions along the line profile as was done by Stenflo (1973) and Frazier and Stenflo (1977), it can be verified that the deviation from unity of the line ratio is really due to the presence of strong fields and not a result of a large difference in ϵ for the two lines.

In conclusion we want to stress that the case we have just discussed indicates the maximum error rather than the probable error, since there will be a strong tendency for source-function equality within the multiplet, whereas we discussed the case when the two lines are formed quite independently of each other. Accordingly we conclude that the line-ratio method gives reliable results when the line pair is carefully chosen, e.g. for the 525.0 and 524.7 nm lines. Strong lines originating higher in the atmosphere are harder to use, since the lower density reduces the close collisional coupling between neighbouring levels within the multiplet and makes it more difficult to obtain source-function equality.

3.4. The Line Source Function and its Scattering Term

Before we discuss the magnetic-field effects on the line source function, let us summarize the multi-dimensional effects for the unpolarized case treated by Stenholm and Stenflo (1977). It was found that the source function depends strongly on Q and f

as well as on the horizontal optical thickness of the cylinder (τ_R). Thus the deviation from LTE increases with decreasing Q until this effect saturates. Similarly an increase in f raises the source function when comparing its values at equal optical depths τ_L in the line. There is a strong coupling between the source functions inside and outside the fluxtube in the upper layers of the atmosphere. The horizontal optical depth τ_R is of importance. since when τ_R gets large (> 1), many photons from the hot walls do not reach the cylinder axis, which reduces the influence of ~~scattering~~ ^{wall photons} in the central portions of the fluxtube.

Let us now consider the dependence of the scattering term e in (25) and (16) on magnetic field strength B . This dependence is illustrated in Figure 9, where $\log (e/e_0)$ is given as a function of height h for the nominal line used (which simulates the 525.0 nm line). e_0 means the values of e when $B = 0$. The left diagram of Figure 9 shows how the scattering term increases with increasing B on the cylinder axis. There is also indication of some saturation in the scattering-term ratio when B becomes ≥ 0.2 T. The most prominent feature is the rapid increase of e/e_0 above a height of about 100 km.

This steep rise of e/e_0 is even more prominent at the edge of the cylinder, as shown by the diagram to the right of Figure 9, giving $\log (e/e_0)$ for various values of R/R_0 with B fixed at 0.2 T. A second region deeper in the atmosphere ($-150 \text{ km} \lesssim h \lesssim 100 \text{ km}$) can also be recognized. The variations in this region are generally smaller but more complex and equally important for the lines considered, since much of the line formation takes

place there. The dependence of e on B and R/R_0 can be explained by optical-depth effects caused by the Zeeman splitting. The splitting makes the profiles broader inside the tube as compared with the outside (cf. Figures 1 - 4). The opacity integrated over all frequencies in the line is the same as in the unpolarized case, but the Zeeman splitting makes the absorption and escape probabilities at a given frequency very different as compared with the unpolarized profile outside the tube.

Accordingly the rapid rise of e/e_0 for $h \gtrsim 100$ km can be given the following explanation: The Zeeman-broadened line profile inside the fluxtube absorbs strongly at frequencies at which the line opacity is negligible outside the tube. This leads to an enhanced interaction between S_L inside the fluxtube and the plane-parallel continuum outside, increasing S_L as compared with the non-magnetic case, for which the interaction with the continuum is through the line profile outside the tube. The stronger enhancement at the cylinder edge as compared with the cylinder axis is an effect of the line opacity ^{inside the tube,} ~~in the horizontal direction,~~ since some of the continuum photons are absorbed before they can reach the cylinder axis.

The lower region ($-150 \text{ km} \lesssim h \lesssim 100 \text{ km}$) is characterized by several opposing effects:

A. The Zeeman splitting enhances the influence of the hot walls by decreasing the line-core opacity and thus the horizontal optical thickness (τ_R), so that more of the continuum photons from the walls can reach the axis. This occurs in layers ($\tau_{c_\infty} \gtrsim 1$ or $h \lesssim 0$) where the line radiation outside the fluxtube is small com-

pared with the continuum radiation.

B. The decrease of the line-core opacity due to the Zeeman splitting increases the probability for photon escape in the vertical direction. This effect is significant for all heights considered ($h \gtrsim -150$ km).

C. Photons far from the line center but inside the Zeeman components can more easily escape through the surroundings, since the line opacity is almost negligible at these frequencies outside the fluxtube. This effect should be significant primarily for $h \gtrsim 0$ km, when the continuum optical depth outside the fluxtube is relatively small.

We can now explain the behaviour of the curves of Figure 9 in the region $-150 \text{ km} \lesssim h \lesssim 100 \text{ km}$ in terms of the above-mentioned effects. The increase of e/e_0 with B is primarily due to effect A, i.e. the decreased τ_R increasing the effect of influx from the hot walls. The decrease of e/e_0 when going from center to edge of the cylinder is due to the more efficient escape, which reduces e (effects B and C). As the optical distance from edge to wall is always optically thin, effect A will disappear at the edge.

Let us now turn to the total line source function S_L and examine how the magnetic-field effects depend on Q and f . The behaviour of S_L is illustrated in Figure 10 by giving $\log (S_L/S_{L_0})$ as a function of h for three different line parameters ($Q = 0.01$ and 0.3 , with f nominal, and $\log f = -3.7$, with Q nominal), both at fluxtube center and edge, with the magnetic field kept fixed at $B = 0.2$ T. S_{L_0} refers to the non-magnetic line source function. The scattering-term effects (e) that have

been discussed above are readily seen in Figure 10:

The region above $h \approx 100$ km depends on Q and f in a relatively simple way. Firstly, we find a moderate decrease of S_L/S_{L_0} when Q decreases. The reason is the following: When scattering dominates, a large portion of the source function is due to photons from the plane continuum outside the fluxtube, even when $B = 0$. This tends to reduce the magnetic-field effects, so that S_L deviates less from S_{L_0} when Q is small. Secondly we have the increase of S_L/S_{L_0} with f , which can be explained as follows: With increasing f , the optical-depth separation between the line-forming region (which now is above $h \approx 100$ km) and the level where the plane continuum is formed rapidly increases. In the non-magnetic case, this tends to decouple the line inside the tube from the surrounding continuum. When the field B is increased, however, the line-forming region will see more and more of the plane continuum when looking downwards, due to the lowering of the line opacity by the Zeeman splitting. This enhances the coupling between the line in the fluxtube and the surrounding continuum, causing a rapid increase of S_L with B . This effect will not apply so much when we get down to the region around $h = 100$ km, where the optical-depth separation between the line in the fluxtube and the plane continuum is relatively small also when $B = 0$. In this region the effect C above of photon escape tends to dominate, particularly for small values of Q .

The variations of S_L in the lower region ($-150 \text{ km} \lesssim h \lesssim 100 \text{ km}$) are more complex but can be given a direct interpretation in terms of the Zeeman-splitting effects. When Q decreases or f increases, ϵ will decrease and the e term (scattering term) will

get a dominating influence on S_L . This will tend to increase the variations of S_L/S_{L_0} . We notice in Figure 11 that at fluxtube axis, when Q is relatively large (e.g. $Q = 0.3$), $S_L > S_{L_0}$. When Q decreases, however, the situation is reversed, and S_L becomes less than S_{L_0} when a magnetic field is present. At the edge of the fluxtube, S_L is always $< S_{L_0}$ in these regions, and it decreases monotonically with decreasing Q . This behaviour is explained as follows:

The increase of S_L above S_{L_0} for moderately large values of Q is caused by effect A above, i.e. the enhancement of the photon flux from the hot walls due to the reduction of the opacity by the Zeeman splitting. The later decline of S_L below S_{L_0} when Q becomes small is due to the increased importance of Zeeman-splitting effects on photon escape, primarily effect B above, because S_{L_0} is already controlled by the hot-wall photons for small values of Q . Since effect A is absent at the edge of the fluxtube, we only have the effects of photon escape there, which explains why $S_L < S_{L_0}$ and decreases monotonically with decreasing Q .

The different behaviour of S_L/S_{L_0} for the line with 10 times larger f is due to the much greater ~~importance of effects of horizontal~~ optical thickness for the stronger line. This suppresses the photon escape and makes effect A more prominent. τ_R at the height of formation of the plane continuum increases strongly with f . For our stronger line we have $\tau_{L_R} = 7.34$ at $\tau_{c_\infty} = 1$ when $B = 0$. Even the optical thickness of the 1 km thick transition zone at the edge rises enough to significantly decrease the photon flux from the hot walls when $B = 0$. However, because of effect A a closer coupling between inside and outside

will be reestablished when the field strength is increased, thus enhancing S_L at $R = 99$ km. S_L at the cylinder axis tries to do the same, but since τ_{LR} remains quite large when B is increased, effect B of photon escape will tend to dominate over the influx effect A .

Let us finally consider the influence of the various Stokes parameters on the scattering term e . Let us by e_Q mean the term in e in (16) that contains the Stokes Q parameter. Similar notations are used for the other Stokes parameters. Thus e_V describes the circular-polarization part of e , $e_Q + e_U$ gives the linear-polarization part, and e_I the intensity part. Figure 11 illustrates the relative contribution of these various parts as a function of height h . The nominal line at the fluxtube axis is used. Variations from center to edge of the fluxtube may be up to 0.5 dex, but the shape of the curves change little with R . For the line with 10 times larger f , the linear and circular polarization curves are raised by approximately 0.5 dex, also without changing the general shape. This increase with f of the relative importance of the polarized part of the radiation is caused by the increased contribution of the line opacity relative to the field-independent continuum opacity.

It should be noted that Figure 11 gives the absolute values, but that the contributions from both the linear and circular polarizations are negative and thus decrease the magnitude of the scattering term e . Note also that the polarization contributions are generally less than 10 % of e . The linear polarization contributes less than the circular polarization except for $h \lesssim 0$ km

when B is large. The increase of the linear polarization with decreasing height h has to do with the increased difference between the source functions inside and outside the fluxtube when h decreases. The contribution to the linear polarization $e_Q + e_U$ comes mainly from the radial direction, while the circular-polarization part e_V prefers the vertical direction, as shown by (7).

The behaviour of e_V/e around $h = 300$ km, where it has a maximum, is explained as follows: Above $h = 300$ km the fluxtube is transparent, and the solid angle occupied by the fluxtube cross section at $h = 0$ (the continuum level outside the fluxtube) is quite small. Therefore the incident continuum photons in the quasi-vertical direction come largely from the outside, so that e_V is determined mainly by the vertical gradients outside the tube. At lower levels, when the tube becomes more opaque and the fluxtube at $h = 0$ occupies a larger solid angle, e_V is determined primarily by the vertical gradients inside the fluxtube. Around $h = 300$ km the e_V/e curve has a transition between these two regions.

Figure 11 further shows that the linear and circular polarization parts increase with the field strength B but saturate at a field of about 0.15 T and 0.1 T, respectively. This saturation is analogous to the saturation of the V profiles in Figures 1 - 4 for $B \gtrsim 0.1$ T. Let us also note that the contributions from both the linear and circular polarizations go to zero at large depths in the atmosphere. This is caused by the increasing dominance of the unpolarized continuum radiation deeper down.

When Q decreases (not shown in Figure 11), $|e_Q + e_U|$ and $|e_V|$ show a slight decrease, but deep in the atmosphere the effect on e_V becomes more pronounced. For small values of Q , e_V even changes sign and becomes positive deep in the atmosphere. The general decrease of $|e_Q + e_U|$ and $|e_V|$ when Q decreases is explained as follows: Scattering couples different regions of the atmosphere and tends to wash out gradients in the source functions. As $|e_Q + e_U|$ depends on the radial gradients and $|e_V|$ on the vertical gradients, both quantities will tend to decrease with an increasing proportion of scattering (decreasing Q).

The sign of e_V is mainly determined by the relative magnitudes of S_L and $(I_+ + I_-)/2$, where I_+ and I_- are the outward and inward intensities in the quasi-vertical direction. Normally S_L is less than $(I_+ + I_-)/2$, which corresponds to $e_V < 0$. When Q is small, however, S_L will be raised due to influx of non-local continuum photons from the hot walls by such an amount that it may exceed $(I_+ + I_-)/2$, resulting in a positive value of e_V .

3.5. Error in the Calculated Stokes Vector when the Non-magnetic Source Function is Used

It has been shown by Rees (1969) that it is generally a very good approximation to use the non-magnetic line source function in treating line formation with incoherent scattering in a magnetic field. This conclusion has been tested for our non-LTE, multi-dimensional model by calculating the emergent Stokes vector in the vertical direction using S_L for $B = 0$ (which gives I_0 and

V_0) as well as using the proper S_L for the actual B (which gives I and V). Table 1 gives I/I_0 and V/V_0 for varying field strengths, frequencies, and line parameters, both at fluxtube axis and edge. The accuracy of the values in the table is about 1 % or better.

The error in I when using the non-magnetic source function is $\lesssim 1$ % for all field strengths and frequencies in the case of the three weaker spectral lines. The V profile can deviate by up to 5 %, although a more common value is 2 %.

For the stronger line the error may exceed 10 % in I and V . The error generally increases when going from axis to edge of the fluxtube, as expected (see Section 3.4). When $B \lesssim 0.025$ T the error is $\lesssim 1$ % for all cases considered.

4. Concluding Remarks

In our earlier paper (Stenholm and Stenflo, 1977) we pointed out three main limiting assumptions used in the calculations:

1. The assumption of a two-level model atom, neglecting non-LTE coupling to other levels.
2. Disregard of Zeeman splitting and polarization.
3. Disregard of velocity-field effects.

In the present paper we have removed the second assumption. The calculations have shown how the line source function varies with field strength. These changes are basically caused by the influence of the Zeeman splitting on the line-opacity structure. However, as can be seen from Table 1 and Figures 1 - 4, the effect on the emerging Stokes vector caused by the source-function dependence on field strength is considerably less important than the primary multi-dimensional effects. This means that the conclusions in our

previous paper, where magnetic-field effects on the source function were disregarded, are still valid.

In many cases it is actually a very good approximation to use the line source function valid for zero magnetic field instead of the correct source function. If we accept a 2 % error in the emerging Stokes vector, this approximation can be used in the case of photospheric lines either if $Q/f \gtrsim 4000$ for all values of B , or for $B \lesssim 0.05$ T for all values of Q and f .

The calculations show that magnetic-field measurements using the line-ratio method of simultaneous magnetograph recordings in two spectral lines are insensitive to the model parameters if the line pair is properly chosen. This can be contrasted with single-line observations, which are very model dependent.

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Table 1. I/I_0 and V/V_0 , where Stokes I and V are calculated in the vertical direction using the proper source function, while I_0 and V_0 are based on the non-magnetic line source function.

Line	R (km)	B (T)	I/I_0 for $\Delta v/\Delta v_D =$				V/V_0 for $\Delta v/\Delta v_D =$		
			0	1	2	3	1	2	3
Weak $Q = 0.3$	0	0.1	1.01	1.01	1.00	1.00	0.99	0.99	0.99
	0	0.2	1.00	1.00	1.00	1.00	0.99	0.98	0.99
	99	0.1	1.00	1.00	1.00	1.00	0.99	0.99	1.00
	99	0.2	1.00	1.00	1.01	1.00	0.99	0.98	0.99
Weak $Q = 0.1$	0	0.1	1.00	1.00	1.00	1.00	0.99	0.99	0.99
	0	0.2	1.00	1.00	1.00	1.00	0.98	0.98	0.98
	99	0.1	1.00	1.00	1.00	1.00	0.99	0.99	1.00
	99	0.2	1.00	1.00	1.01	1.00	0.99	0.98	0.99
Weak $Q = 0.01$	0	0.1	1.00	1.00	1.00	1.00	1.00	0.99	0.98
	0	0.2	1.00	1.00	1.00	1.00	1.01	1.00	1.00
	99	0.1	1.00	1.00	1.00	1.00	1.01	1.02	1.05
	99	0.2	1.00	0.99	1.00	1.00	1.05	1.02	1.05
Strong ($\log f =$ -3.7) $Q = 0.1$	0	0.1	1.05	1.01	1.01	1.00	0.97	0.99	1.00
	0	0.2	1.00	1.03	1.05	1.04	0.95	0.94	0.94
	99	0.1	1.10	1.03	1.02	1.01	0.98	0.97	0.93
	99	0.2	1.06	1.08	1.09	1.08	0.87	0.88	0.89

Figure Captions

Fig. 1. Stokes I and V profiles normalized to the local continuum intensity I_c , for the line with $Q = 0.3$ and $\log f = -4.7$. The four diagrams represent field strengths 0.025, 0.05, 0.1, and 0.2 T.

Solid lines: non-LTE profiles at fluxtube axis.

Dashed lines: LTE profiles at fluxtube axis.

Dotted lines: non-LTE profiles far outside the fluxtube, and, in the case of the diagram to the lower right, also the LTE profile far outside the fluxtube.

Fig. 2. Same as Figure 1, but for the line with $Q = 0.1$ and $\log f = -4.7$. In addition, we have drawn the profiles at the fluxtube edge at $R/R_0 = 0.99$ (dashed-dotted), when they differ significantly from the profiles at fluxtube axis (which is not the case for the line in Figure 1).

Fig. 3. Same as Figures 1 and 2, but for the line with $Q = 0.01$ and $\log f = -4.7$.

Fig. 4. Same as Figures 1 and 2, but for the line with $Q = 0.1$ and $\log f = -3.7$.

Fig. 5. Ratio between the apparent field strength $\bar{B}_{\text{obs}}$ and the true field strength B as a function of B for our weaker spectral line with four different values of the collisional-excitation cross section Q . The three curves in each diagram represent different positions in the line profile.

Fig. 6. Same as Figure 5, but for our stronger spectral line with 10 times larger f ($= 2.0 \times 10^{-4}$) and for two values of Q .

Fig. 7. Ratio between the apparent field strengths observed in a line with a Landé factor $g = 3.0$ ($\bar{B}_{\text{obs}_3}$) and a line with $g = 2.0$ ($\bar{B}_{\text{obs}_2}$) for our weaker spectral line and different combinations of Q for the two lines. Thus $(Q = 0.1)/(Q = 0.3)$ denotes that $Q = 0.1$ for the $g = 3$ line and $Q = 0.3$ for the $g = 2$ line. The different curves in each diagram refer to true field strengths $B = 0, 0.05, 0.1, 0.15$, and 0.2 T. The solid lines are the same in all diagrams and refer to the case $(Q = 0.1)/(Q = 0.1)$. This case practically coincides with the LTE case, i.e. $(Q = \infty)/(Q = \infty)$.

Fig. 8. Same as Figure 7, but for our stronger spectral line with 10 times larger $f(= 2.0 \times 10^{-4})$.

Fig. 9. $\log(e/e_0)$ as a function of height h , where e is the scattering term (16) in the source function, and e_0 is the value of e when $B = 0$. The diagram to the left is valid at the fluxtube axis and shows curves for different values of the field strength B . The diagram to the right has the field strength fixed at 0.2 T, and the curves represent different positions within the fluxtube. The heights where the line and continuum optical depths are unity far outside the fluxtube (index ∞) and at the fluxtube axis (index 0) are indicated. As the height where $\tau_{L_0} = 1$ varies with field strength, the left diagram gives only the range of heights where this condition is satisfied.

Fig. 10. $\log (S_L/S_{L_0})$ as a function of height h , where S_L is the line source function (25), and S_{L_0} is the value of S_L when $B = 0$. The curves represent three different spectral lines: the stronger line with $Q = 0.1$, the weaker line with $Q = 0.3$ and 0.01 . Solid lines: Fluxtube axis. Dashed lines: Fluxtube edge. The heights of unit optical depths are indicated as in Figure 9.

Fig. 11. Relative contributions (as defined in the text) of the various Stokes components to the scattering term e in (16). The curves are valid for the fluxtube axis and represent different values of the field strength B . The heights of unit optical depths are indicated as in Figure 9.

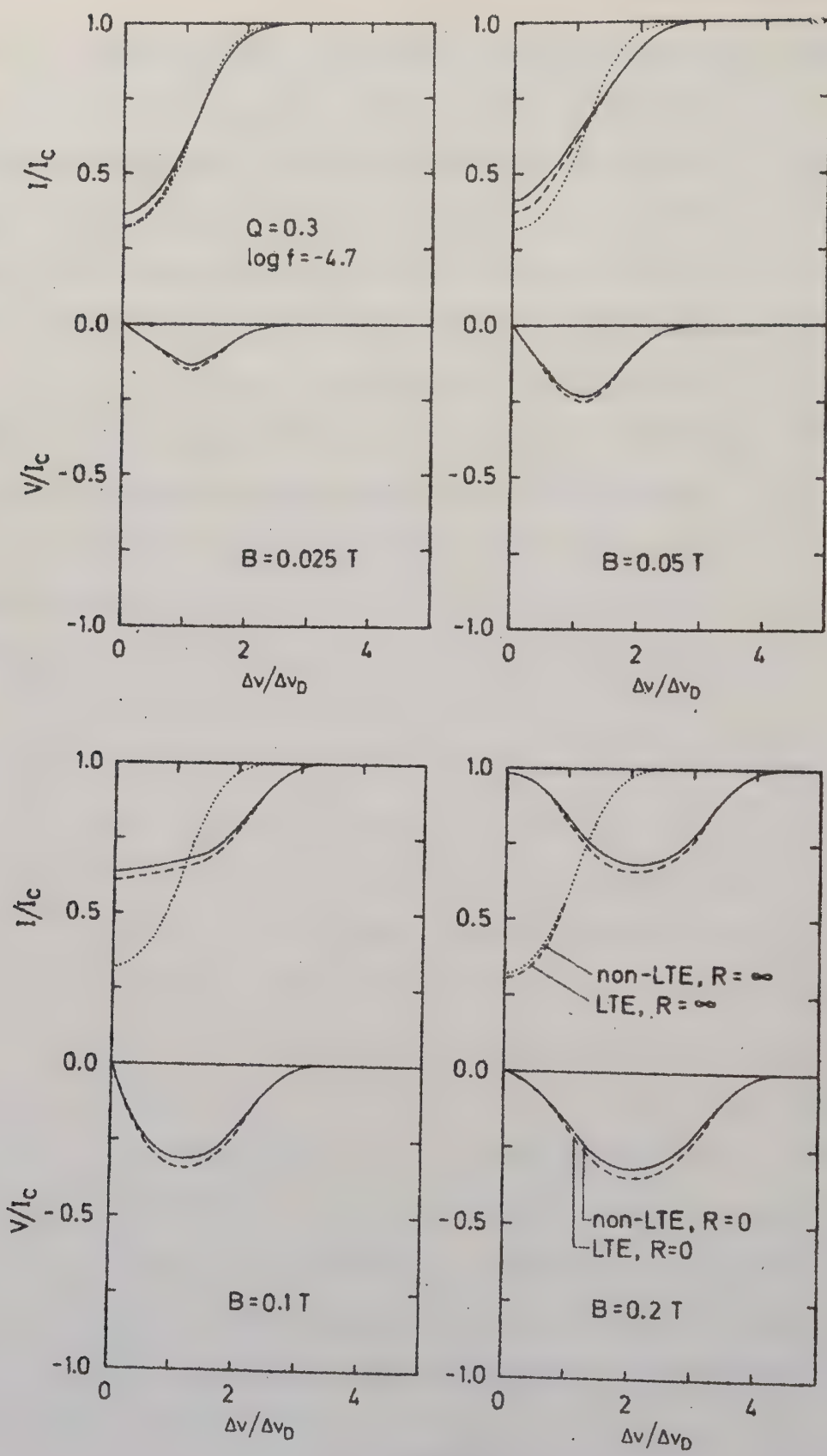


Fig. 1

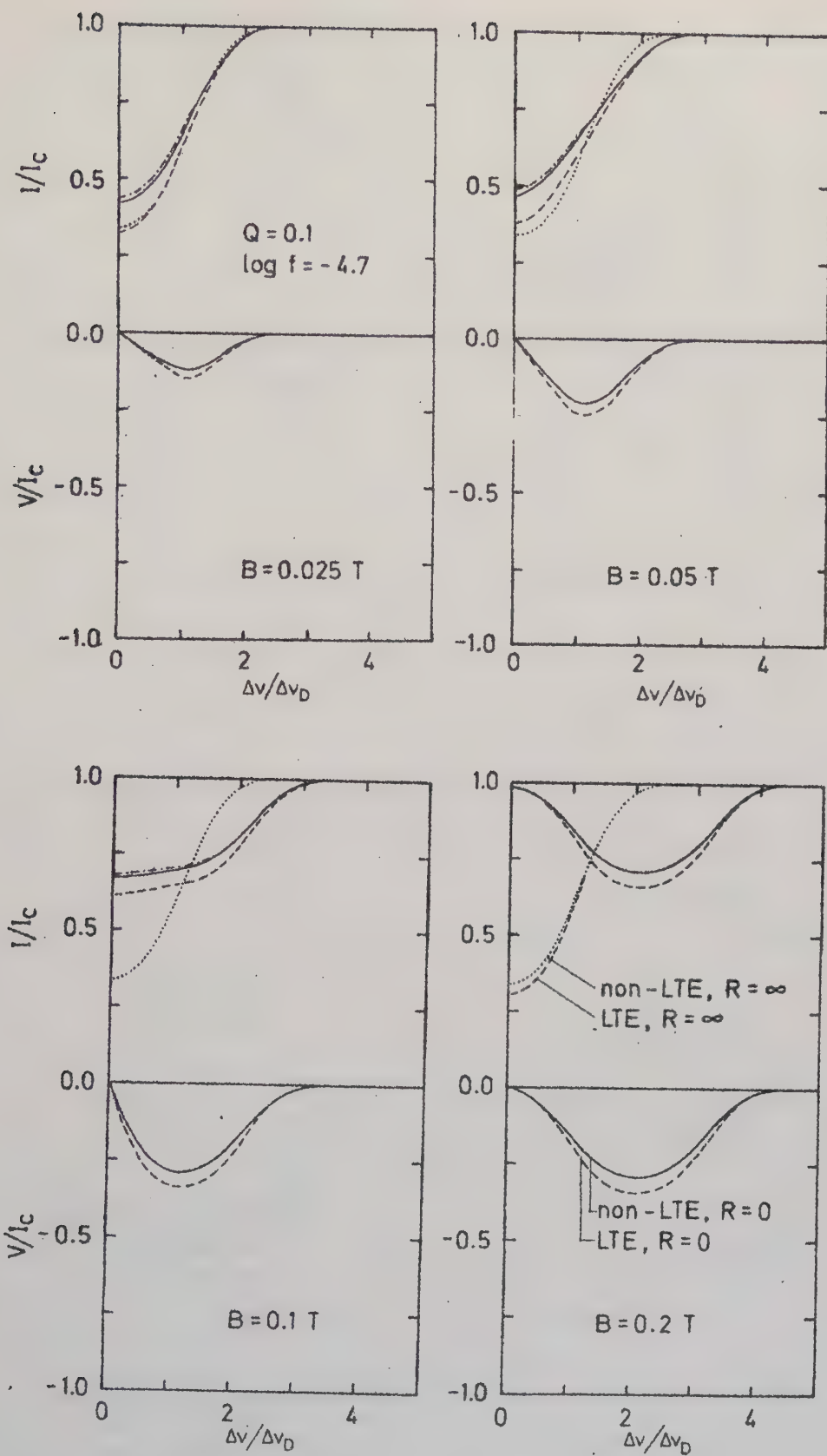


Fig. 2

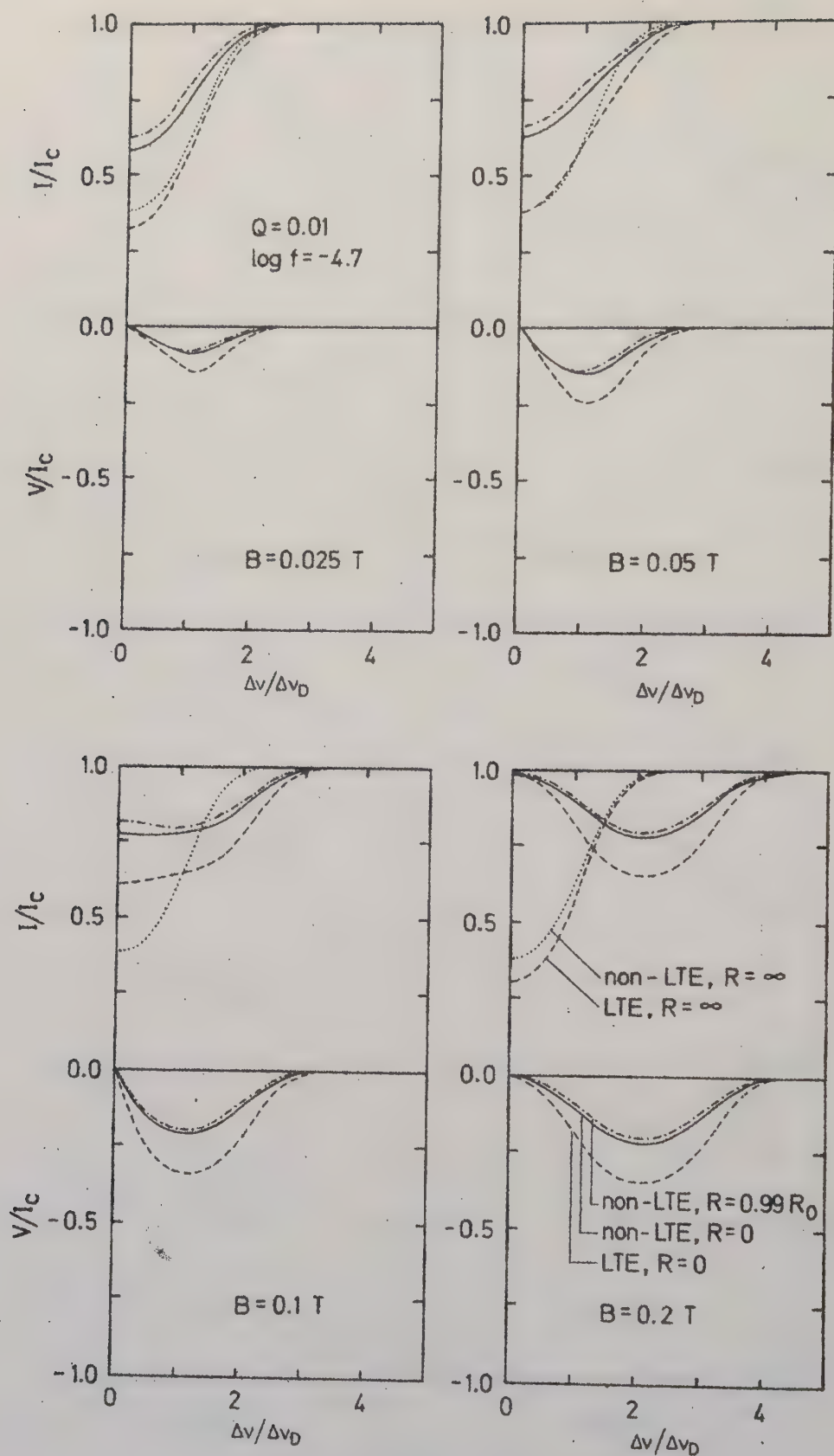


Fig. 3

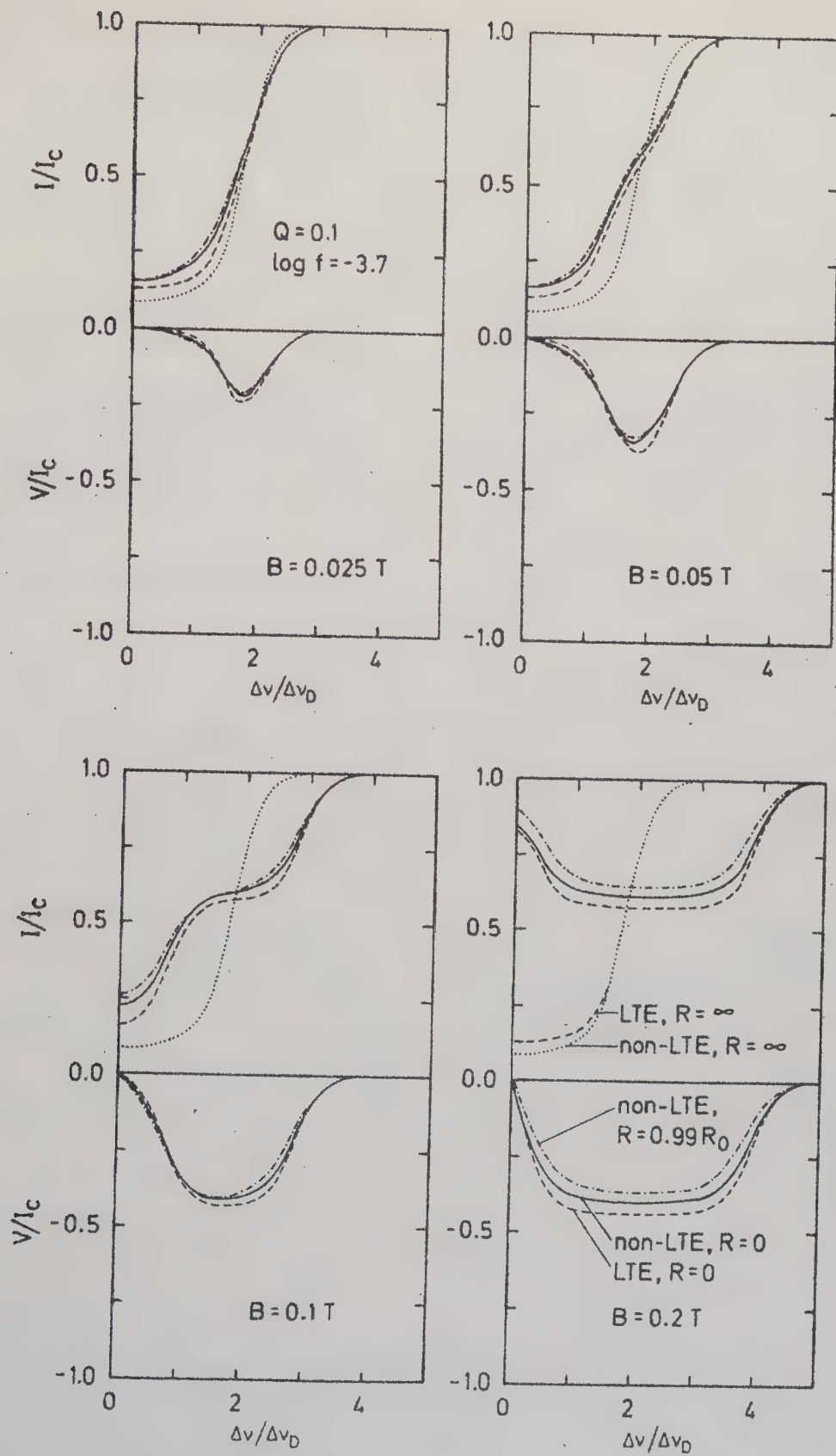


Fig. 4

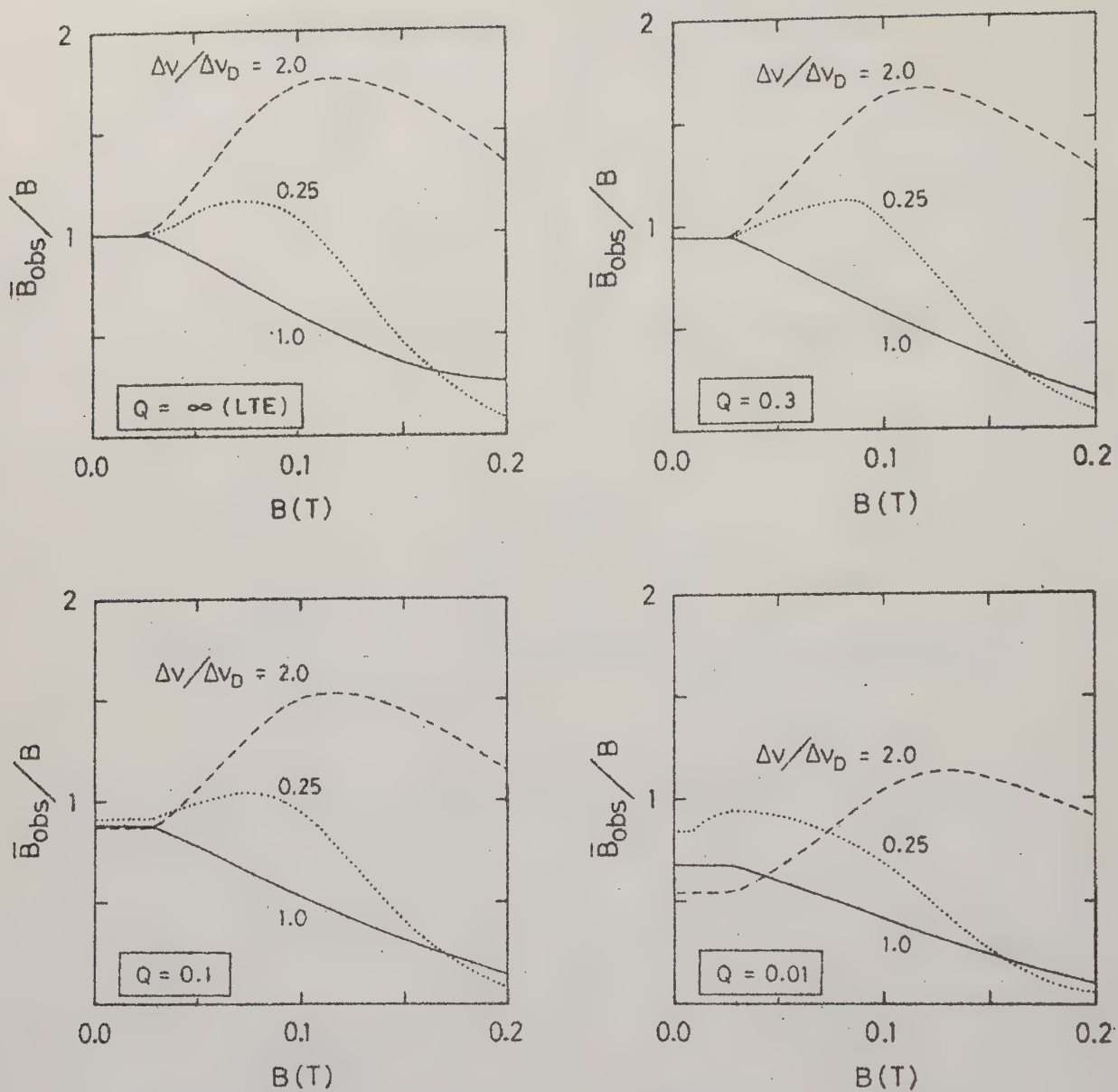


Fig. 5

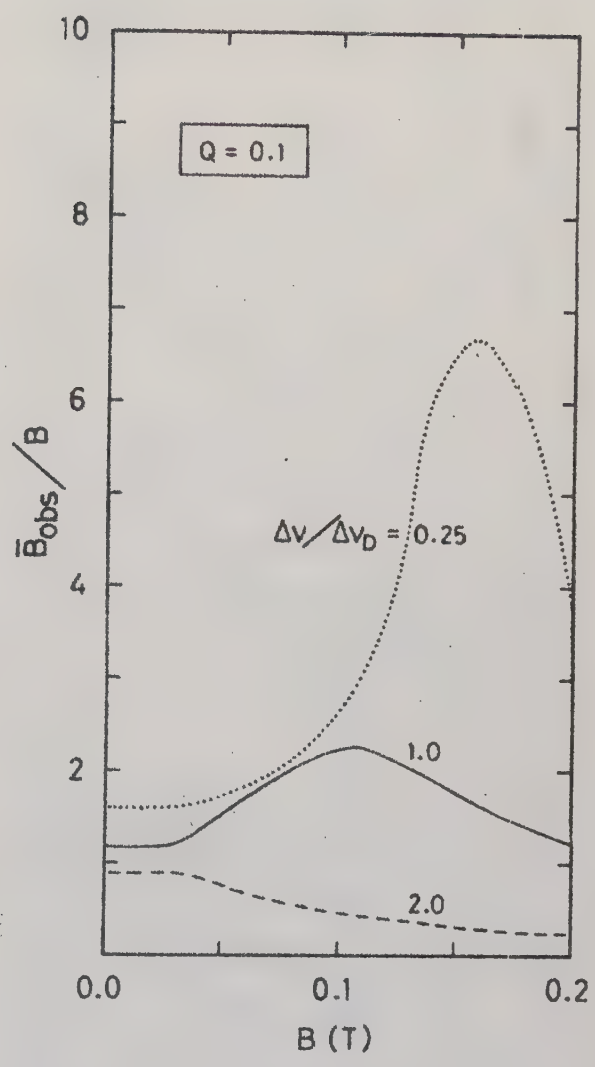
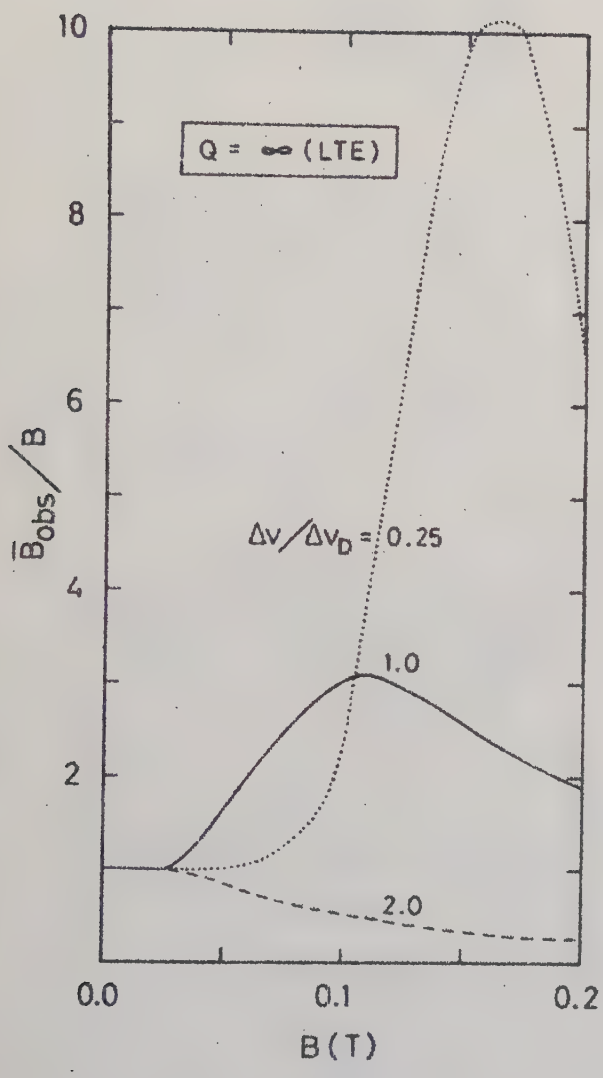


Fig. 6

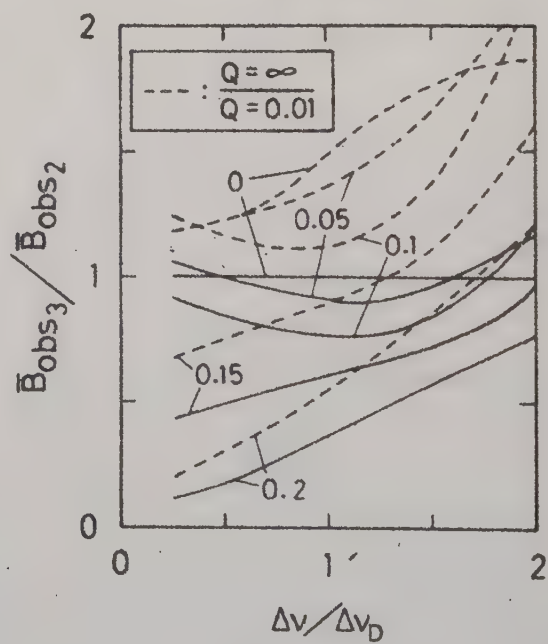
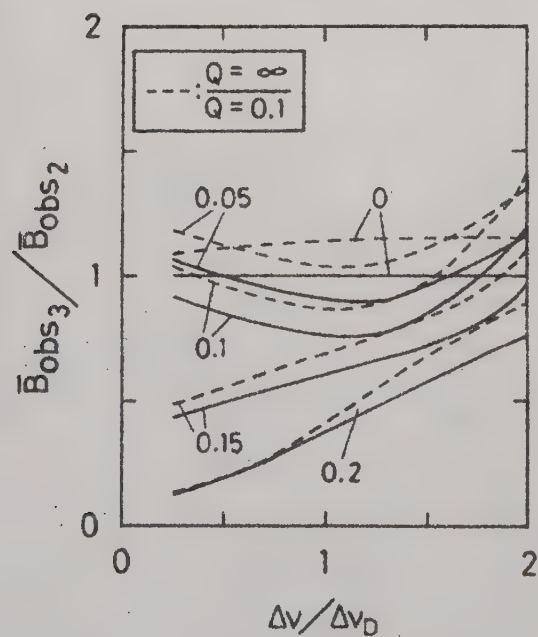
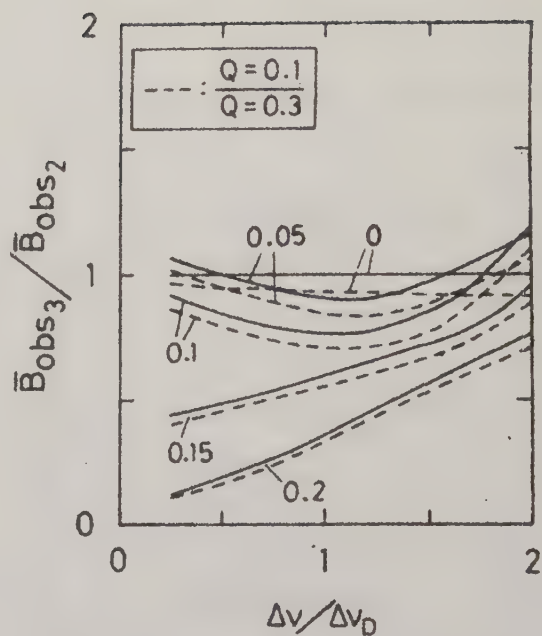
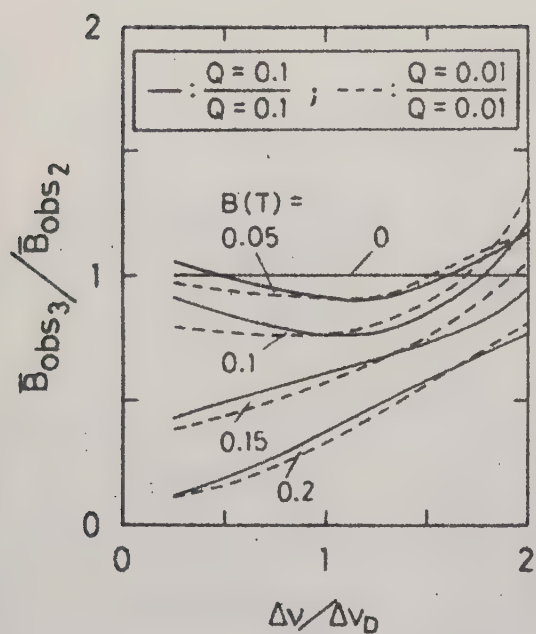


Fig. 7

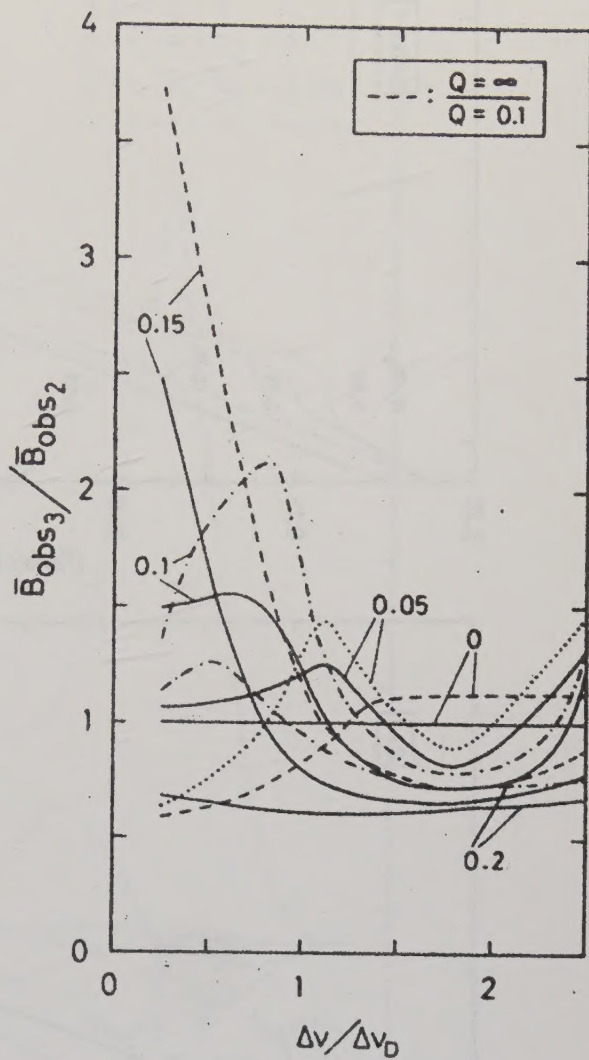
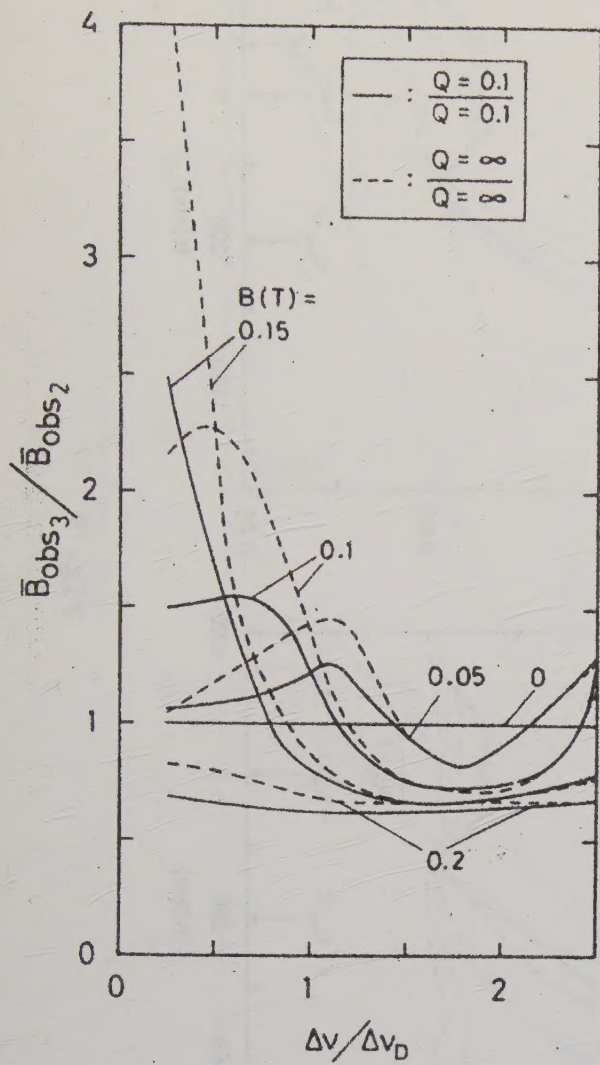


Fig. 8

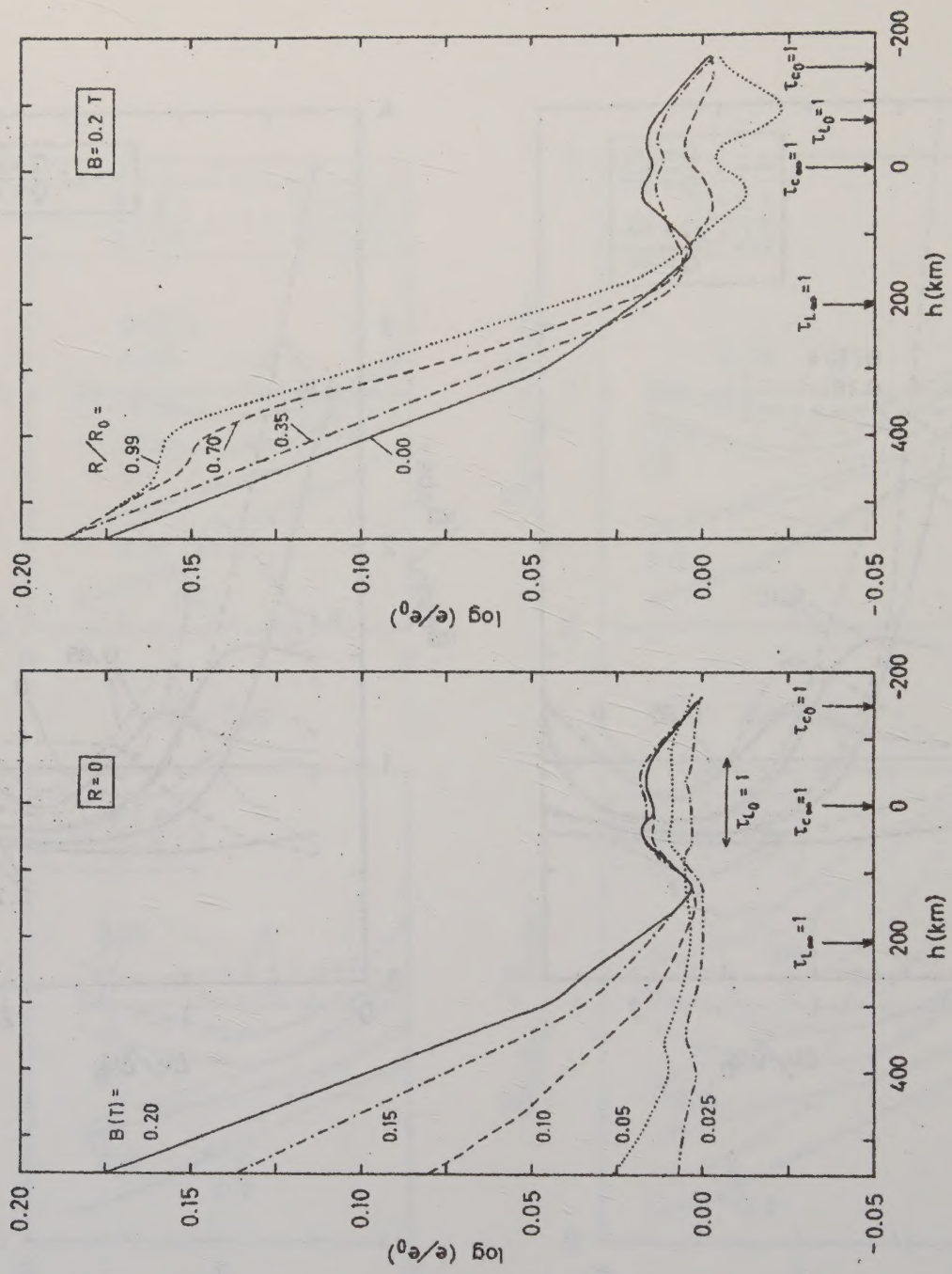


Fig. 9

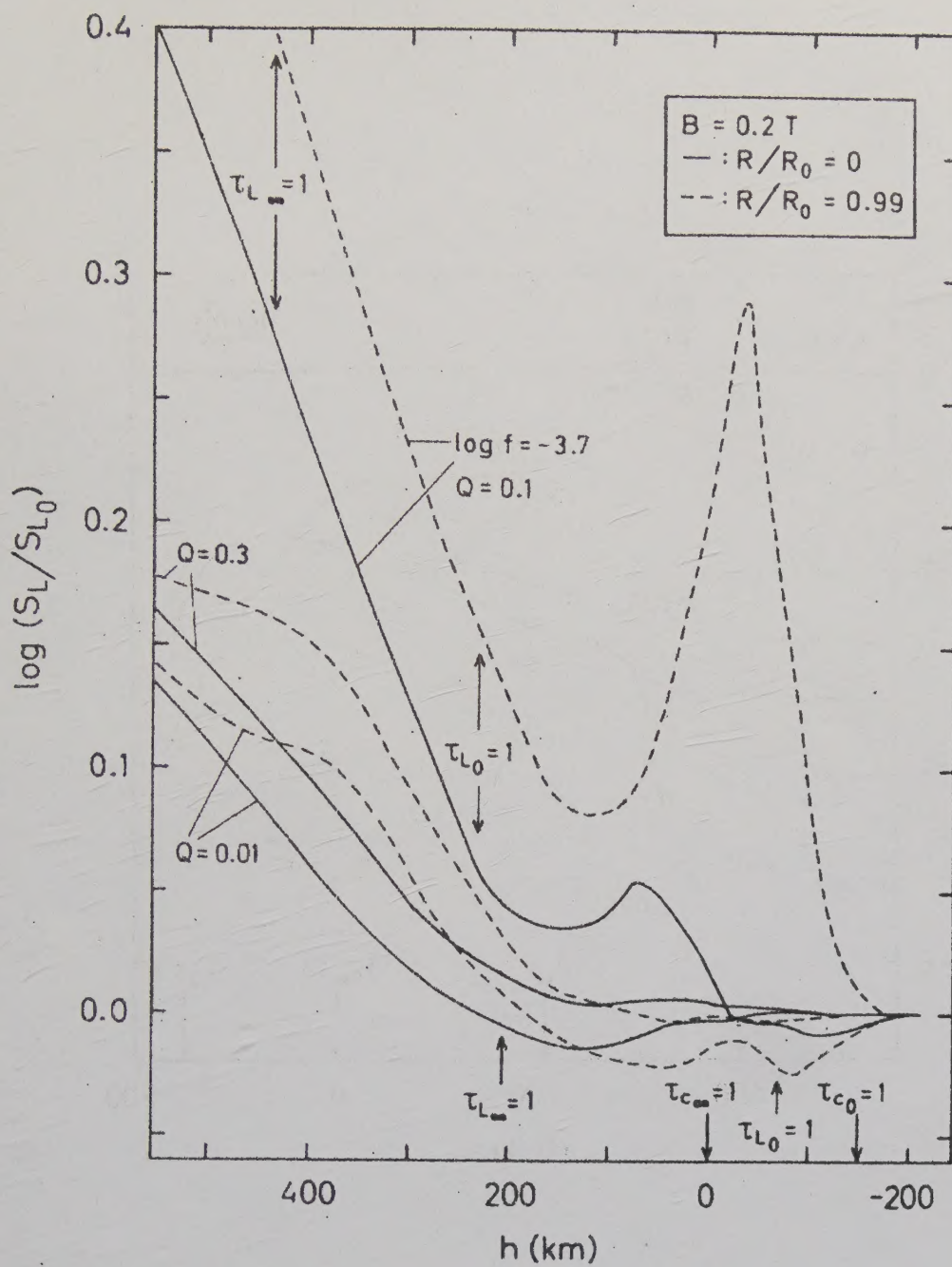


Fig. 10

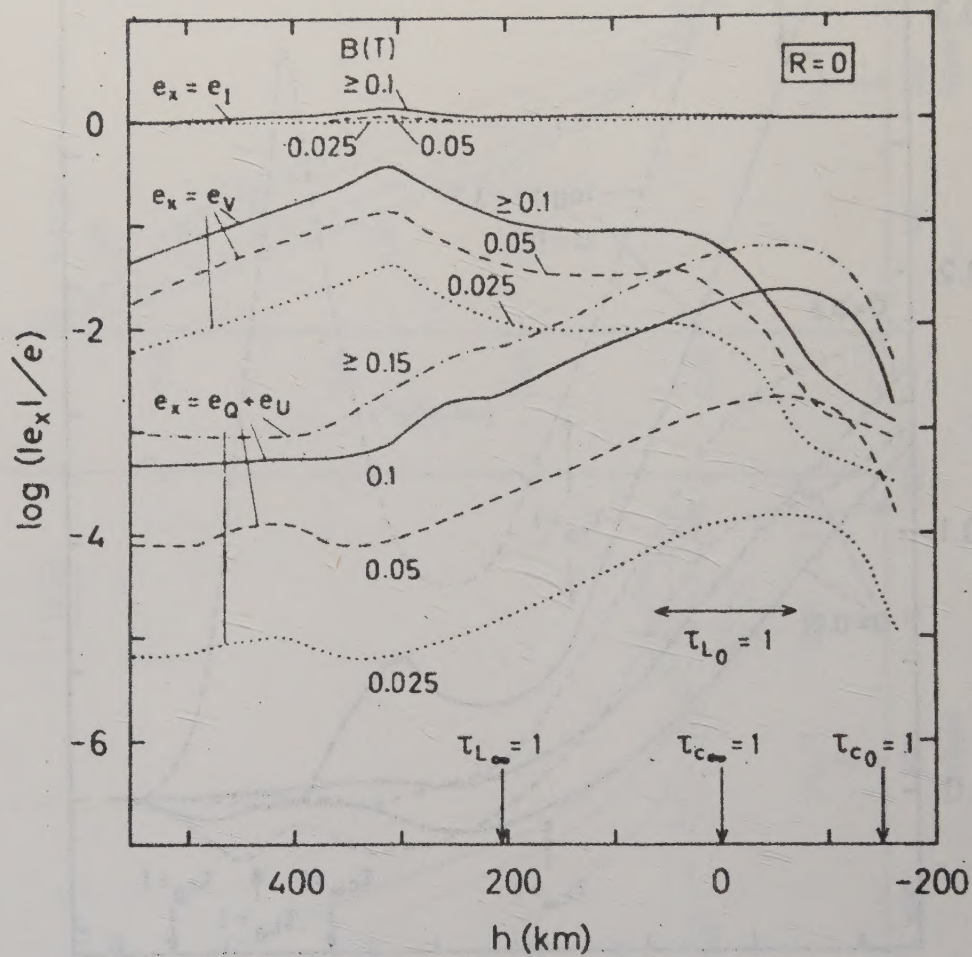


Fig. 11